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RESEARCH ARTICLE

Cylindrical Gears with Changing Ratio

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Abstract
Changing ratio gears are noncircular. It means that if they are combined with a conventional cylindrical gear a changing axis distance will be given. Changing axis distance can generate a lot of difficulties and additional costs in the construction. Our goal was to create changing ratio gears that have cylindrical geometry and interlock with conventional cylindrical gears at a constant axis distance. This can be achieved by applying continuously changing profile shift. Profile shift modifies the diameter of the gears, however it does not have any effect on the ratio; regardless if that is constant or changing. The point of our calculation is that some gears with noncircular rolling curves can have circular pitch curve with the help of changing profile shift.

Keywords
gears, noncircular gears, changing profile shift, changing ratio, changing axis distance, envelope

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1 Introduction

Changing ratio noncircular gears are rare machine parts; however the application of these elements offers many possibilities. There are some special fields of engineering where these solutions can have numerous advantages compared to the classical, conventional solutions. With the help of these gears previously defined angular rotation functions or torque functions are relatively easily implementable. They can be applied for example in special steering mechanisms, balancing mechanisms, or in special flow measure equipments Fig. 1 [3, 8, 10]. Of course there can be a lot more application.

These special gears can have opened or closed-loop pitch curves and the axis distance can be either constant or changing. Noncircular gears with constant axis distance and closed pitch curves are generally used to create or eliminate alternating angular velocities or torques. The pitch curve calculation of these gears is described in the next chapter.

It is important to mention that the ratio alternation of these pitch curves repeats after at least one whole revolution of the smaller gear. This means that the cog count ratio has to be an integer. If such gears are needed which don't have this kind of cog count ratio, the axis distance will alternate. This makes both the calculation and the construction a lot more complicated. The pitch curves of these gears can be determined with the help of the rigid body velocity equations [1, 5], where not just the angular velocities but the axis distance is as well a function.

In this article however we introduce a method that is suitable for creating cylindrical gears with changing ratio with the help of a continuously changing profile shift. The resulting gear is cylindrical and has a constant axis distance paired with a conventional cylindrical gear but the ratio is continuously changing.

2 Pitch curves of noncircular gears

The (rolling) pitch curves of a pair of noncircular gears can be defined by an a constant axis distance, which is the sum of the working radii $a = r_1 + r_2$ and transmission function, which is given by Eq. (1).

$$i\left(\varphi_2\left(\varphi_1\right)\right)=\frac{\omega_1}{\omega_2}=\frac{\frac{d\varphi_1}{dt}}{\frac{d\varphi_2}{dt}}=\frac{d\varphi_1}{d\varphi_2} \quad (1)$$

where ω_i is the angular velocity, φ_i is the angle position of the i^{th} gears. In Fig. 2(a), the kinematic chain of the two non-circular gears is presented. The contact point is placed on the line between the axes. The gears have the same velocity vector in this point, so Eq. (2) is valid.

$$r_1\omega_1=r_2\omega_2 \quad (2)$$

Hence, the transmission can be defined by the working radii and/or the axis distance Eq. (3).

$$i=\frac{\omega_1}{\omega_2}=\frac{r_2}{r_1}=\frac{a-r_1}{r_1}. \quad (3)$$

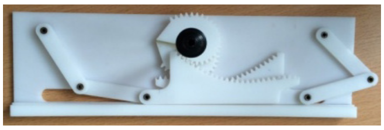
Rearranging Eq. (3), the first rolling curve in polar coordinate system can be defined as Eq. (4).

$$r_1\left(\varphi_1\right)=\frac{a}{i\left(\varphi_1\right)+1} \quad (4)$$

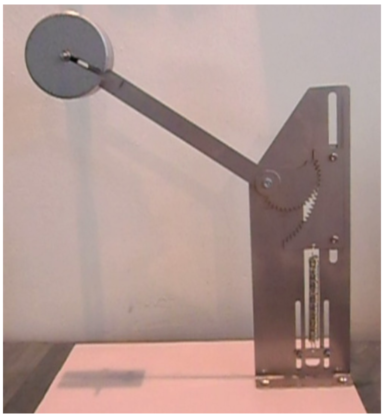
The rolling curve of the second gear can be defined as Eq. (5).

$$r_2\left(\varphi_2\left(\varphi_1\right)\right)=r_1\left(\varphi_1\right)i\left(\varphi_1\right) \quad (5)$$

a)

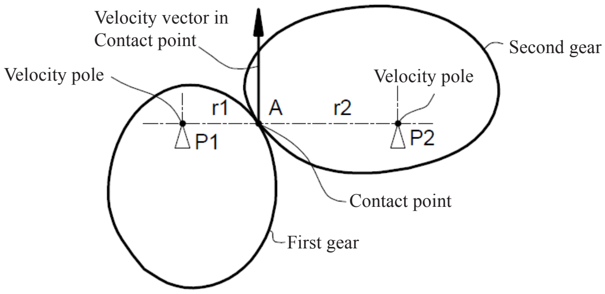


b)



c)

Fig. 1 Application of noncircular gears: a) kinematic mockup of a special steering mechanism [3], b) Weight balancing mechanism [8], c) Water flow measure equipement [10].



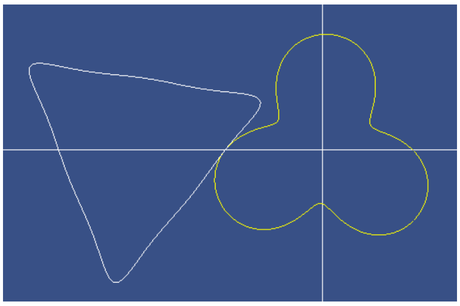


Fig. 2 Pitch curves of noncircular gears: a) Sketch for the evaluation of the curves, b) An example pair of pitch curves [8]

Applying these calculations we created a pair of noncircular gears that eliminates the angular velocity alternation of a 45° deflected cardan joint (Fig. 3). This could be one of the many applications of this kind of gears [8].

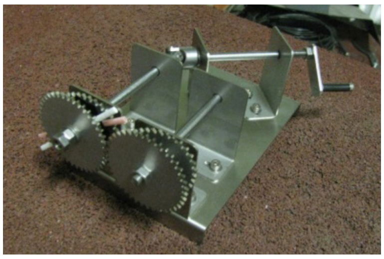


Fig. 3 Device that eliminates the angular velocity alternation of a 45° deflected cardan joint [8].

3 Relation between profile shift and pitch curve slip

It is known that the axis distance of cylindrical gears can be slightly modified by applying profile shift [7, 9]. This modification does not have any influence on the ratio. Our assumption is that continuously changing profile shift a slightly non-circular gear can be modified to a cylindrical one without having any effect on the noncircular rolling curve. This means that the resulting special gear will be able to operate with a conventional cylindrical gear at a constant axis distance.

The calculation that we worked out are based on the Fellows's method of gear manufacturing [7, 9]. Accordingly we studied first the process of a conventional constant profile shift.

Using the marking of Fig. 4, we want to generate a gear with a pitch circle radius $r_1=\frac{m\cdot z_1}{2}$. For this we use a cutting gear with a pith circle radius $r_2=\frac{m\cdot z_2}{2}$ and a profile shift of $x\cdot m$. In these expressions m is the module, z_1 and z_2 are the number of teeth. For the virtual gear generating the r_2 circle of the cutting gear has to be rolled down on a circle of $r_0 = r_1 + x \cdot m$ with a certain slip. For this the following transformation steps have to be carried out on each points of the cutting gear [6]:

- Rotation with ψ angle around the centre of the cutting gear
- Translation in Y direction with $r_0 + r_1$
- Rotation with φ angle around the origin

Based on Fig. 4 the following Eq. (6) equations persist.

$$\frac{\varphi}{\psi} = \frac{r_2}{r_1} = \frac{z_2}{z_1} = i \;\Rightarrow\; \varphi = \psi \frac{r_2}{r_1} = \psi \frac{z_2}{z_1} \tag{6}$$

If we substitute the expression $r_1 = r_0 - x \cdot m$ in Eq. (6), we get Eq. (7).

$$\varphi = \psi \frac{r_2}{r_0 - x \cdot m} \tag{7}$$

The expression Eq. (7) creates a relation between profile shift and the rotation angles of the gears. It means that the profile shift correlates with the slip on the pitch circle. In order to make this equation easier to comprehend and use, we transpose it in the following format (Eq. (8)).

$$\varphi(\psi) = \psi \frac{r_2}{r_0 - x \cdot m} = \varphi_0(\psi) + \varphi_x(\psi) \tag{8}$$

In Eq. (8) $\varphi_0(\psi)$ represents the linear angular rotation function of the cutting gear without any profile shift. $\varphi_x(\psi)$ is a linear function representing the slip that generates the profile shift. Rearranging Eq. (7) and Eq. (8) we get the functions mentioned before (Eq. (9)).

$$\varphi_0(\psi) = \psi \frac{r_2}{r_0},\ \varphi_x(\psi) = \frac{m \cdot x \cdot \psi \cdot r_2}{r_0(r_0 - m \cdot x)} \tag{9}$$

With this Eq. (9) we defined a $\varphi_x(\psi)$ linear function that creates relation between x profile shift parameter and the angular slip of the cutting gear on a given r_0 pitch circle (Fig. 4).

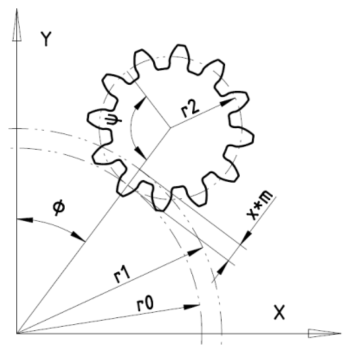


Fig. 4 Sketch used for the coordinate transformation

4 Changing profile shift

The major point of the described method is that this x profile shift parameter is not necessarily a constant but can also be a $x(\psi)$ function. This means that the instantaneous radius of the rolling circle can be modified by the $\varphi_x(\psi)$ slip angle function of the cutting gear, without modifying the pitch circle radius.

If a closed loop gear with integer number of teeth is needed, the $x(\psi)$ profile shift function has to be defined so that $\varphi_x(\psi)$ slip angle function remains periodic (Eq. (10)).

$$\varphi_x(\psi = 0) = \varphi_x\left(\psi = 2\pi \frac{z_1}{z_2}\right) \tag{10}$$

In order to make the calculation simpler it is recommended to directly define the $\varphi_x(\psi)$ function of slip angle instead of defining the $x(\psi)$ function of profile shift. If needed, this function can be calculated later from the directly given $\varphi_x(\psi)$ function with the help of Eq. (11).

$$x(\psi) = -\frac{\varphi_x(\psi) \cdot r_0^2}{m \cdot \varphi} \tag{11}$$

The $\varphi_x(\psi)$ function of slip angle can be easily generated using the intended ratio function. If the connecting gear's number of teeth equals to the cutting gear's number of teeth Eq. (12) is valid, where $i(\psi)$ is the function of the changing ratio. The function of the ratio can be created analytically or with optimization [12].

$$i(\psi) = \frac{\varphi(\psi)}{\psi} = \frac{\varphi_0(\psi) + \varphi_x(\psi)}{\psi} \tag{12}$$

As seen in Eq. (11), if we intend to increase the absolute maximum of $\varphi_x(\psi)$ slip angle function, the module has to be increased compared to the r_0 pitch circle radius, so the number of teeth has to be reduced. At fewer cogs however the possible profile shift is smaller as well (Fig. 5). This leads to a very curious phenomenon that the number of teeth has a very small effect on the maximal absolute value of the $\varphi_x(\psi)$ slip angle function.

On Figures Fig. 6 changing ratio cylindrical gears are seen, which have angle alternation and different numbers of teeth. At these examples we applied sine function for the angle alternation with amplitude of 3°.

5 Additional constant profile shift

According to our experience applying changing profile shift the maximal value of angle alternation is around ±3° on a 180° range. Exceeding this maximum value the teeth will be undercut and the top land curves will disappear.

These negative effects can be reduced and the angle alternation can be increased by applying constant profile shift or in some case modifying the pressure angle. With these actions it is possible to create angle alternation of around ±6° on a 180° range.

By applying additional constant profile shift the second step of the above described coordinate transformation has to be modified:

The translation in Y direction should be done with $r_0 + r_2 + x_c \cdot m$, where x_c is a constant profile shift parameter (Fig. 7, Fig. 8).

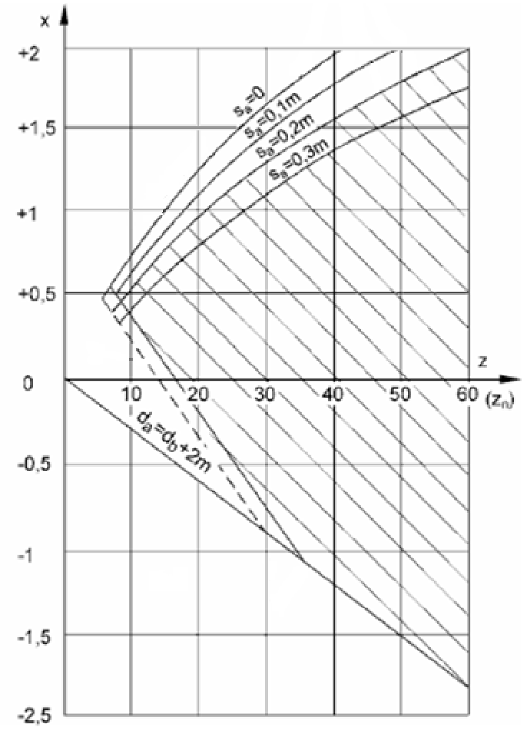


Fig. 5 Graph representing the maximum limits of profile shift in dependence with the addendum tooth thickness and the number of teeth [11]

6 Envelope calculation

After carrying out the coordinate transformations the envelope of the resulting set of lines is to be determined. There are numerous methods for this task [2-4, 8]. We applied a method of searching line chains that is a fast and robust method [3].

The resulting envelopes can be seen on Fig. 9.

7 Conclusion

In this article we demonstrated a method that is capable of generating changing ratio cylindrical gear's geometry using continuously changing profile shift. These special gears can interlock with a conventional cylindrical gear at a constant axis distance. In our calculation the Fellow's method of gear generating was applied, where a cutting gear generates the resulting geometry. After determining the relation between the profile shift and the slip of the pitch curves, the constant profile shift parameter was substituted with a function. In our calculation the alternating profile shift does not have any effect on the pitch curve, only on the rolling curve.

After carrying out the coordinate transformation an envelope finding algorithm was applied, that results the final geometry of the gear.

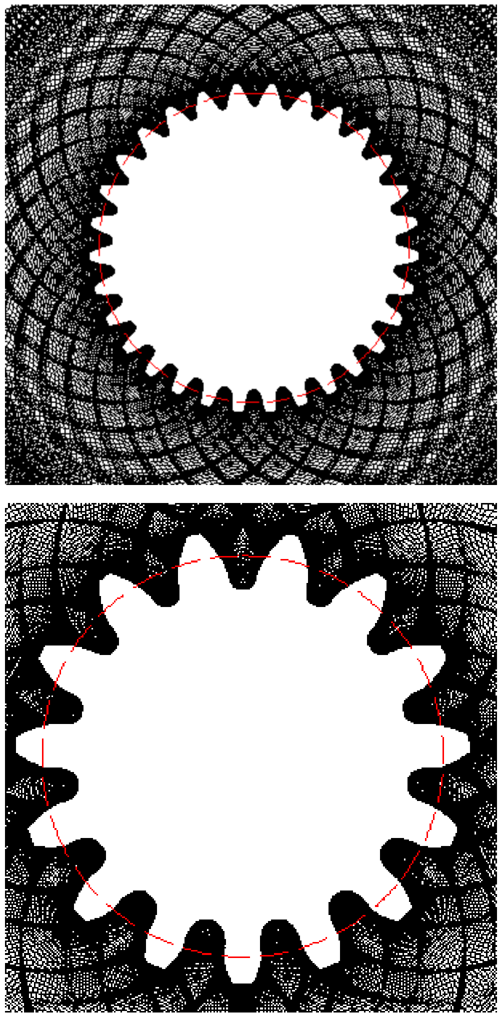


Fig. 6 Changing ratio cylindrical gears with a 3° amplitude sine function like angle alternation and different numbers of teeth.

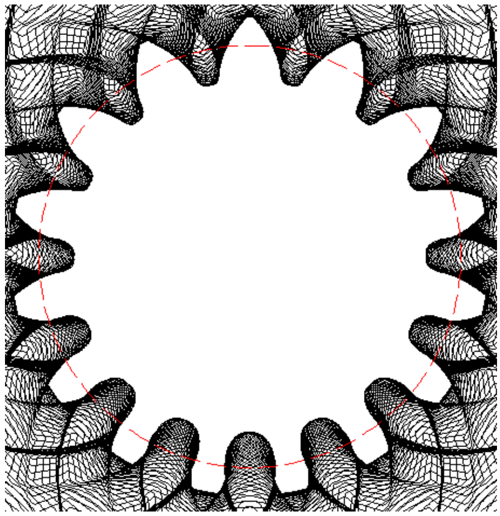


Fig. 7 Changing ratio cylindrical gear with a 6° amplitude sine function like angle alternation, without constant profile shift.

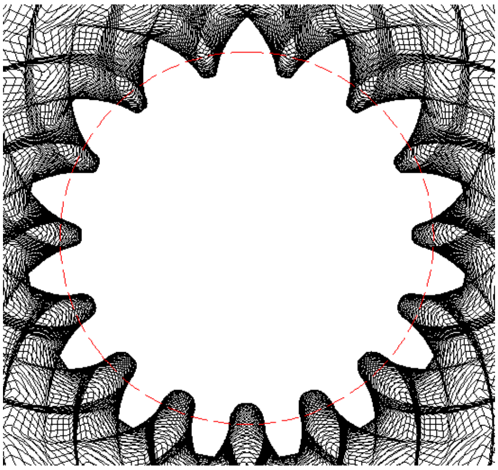


Fig. 8 Changing ratio cylindrical gear with a 6° amplitude sine function like angle alternation, with constant profile shift.

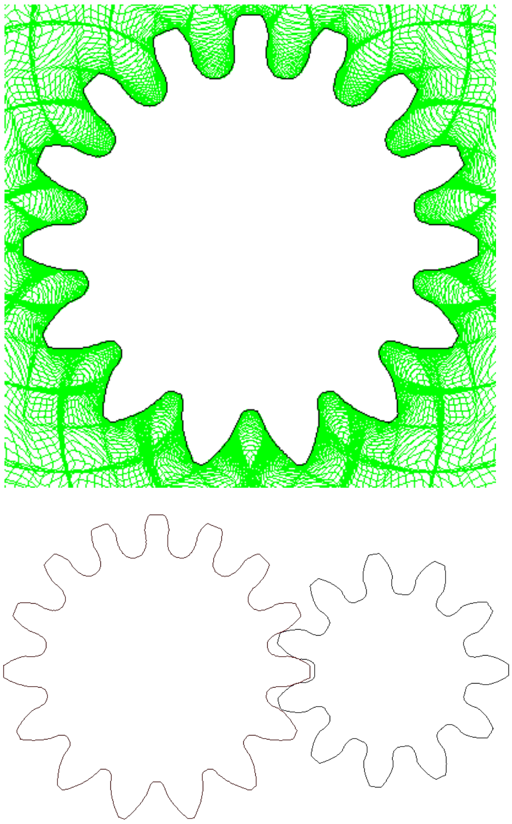


Fig. 9 Resulting envelope of the gears

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